

Breckon Hill Primary School

Maths Calculation Policy



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This document shows progression in calculation (addition, subtraction, multiplication and division) in line with the National Curriculum. The consistent use of the CPA (concrete, pictorial, abstract) approach within our school helps pupils develop mathematics across all the operations in an efficient and reliable way. Our calculation policy supports staff and parents by demonstrating how these methods progress to develop pupils' confidence, proficiency and deepen their understanding of both written and mental methods.

Sound mental fluency underpins written calculations, enabling pupils to make choices about when a written method should be used and when a mental strategy would be most appropriate. Correct modelling of mathematical language is key to helping pupils explain their methods and develop links between mathematical operations.

Intent

Our 5 main aims when teaching mathematics are to develop:

- A solid conceptual understanding and for children to make links between different mathematical concepts.
- Fluency with number.
- Children who see maths as relevant in the real world.
- Positive, enthusiastic attitudes.
- Children who are curious and resilient.

In line with the 2014 National Curriculum the emphasis has been to ensure that all children:

- Become fluent.
- Reason and explain mathematically.
- Can solve problems.

Glossary

Correct modelling of mathematical vocabulary is essential for pupils to explain and understand mathematical structures. The following terms should be introduced as pupils progress throughout the school.

Addend A number to be added to another number. E.g. $7+8= 15$ 7 and 8 are addends.	Aggregation To combine 2 or more quantities or measure to find a total. E.g. Mina has 3 pencils and George has 2, how many pencils are there altogether?	Array An ordered collection of counters, cubes or other items in rows and columns.
Augmentation Increasing a quantity or measure by another quantity. E.g. A plate has 5 cakes, 4 more are added. How many cakes does the plate have now?	Commutative Addition and multiplication are commutative; the equation can be carried out in any order.	Complement In addition, a number and its complement make a total. e.g 300 is the complement to 700 to make 1000.
Difference The numerical difference between two numbers is found by comparing the quantity in each group.	Dividend The number that is divided in a division equation. E.g. $24 \div 6 = 4$ 24 is the dividend.	Divisor In division, the number by which another is divided. E.g. $24 \div 6 = 4$ 6 is the divisor.
Exchange Change a number or expression for another of equal value.	Factor A number that multiplies with another to make a product. E.g. $6 \times 4 = 24$ 6 and 4 are factors of 24	Minuend A quantity or number from which another is subtracted. E.g. $20 - 9 = 11$ 20 is the minuend
Partition To split a number into its component parts.	Product The result of multiplying one number by another. E.g. $6 \times 4 = 24$ 24 is the product	Quotient The result of a division. E.g. $24 \div 6 = 4$ 4 is the quotient.
Remainder The amount left over after a division when the divisor is not a factor of the dividend. E.g. $25 \div 6 = 4 \text{ r}1$ as 6 (the divisor) is not a factor of 25 (the dividend)	Scaling Enlarging or reducing a number by a given amount, called the scale factor	Subitise To instantly recognise a number of objects in a small group without needing to count.
Subtrahend A number to be subtracted from another. E.g. $20 - 9 = 11$ 9 is the subtrahend.	Sum The result of an addition. E.g. $7+8= 15$ 15 is the sum.	Total The sum found by an addition. e.g. $7+8= 15$ 15 is the total

Key Principles

Number Sense		
<ul style="list-style-type: none"> • An awareness of the relationship between number and quantity. • An understanding of number symbols, vocabulary and meaning • The ability to engage in systematic counting, including notions of cardinality and ordinality • An awareness of magnitude and comparisons between different magnitudes • An understanding of different representations of number • Competence with simple mathematical operations • An awareness of number patterns including recognising missing numbers <p>Andrews, Sayers and Back, (2013)</p>	<ul style="list-style-type: none"> • Subitising • Magnitude – knowing which set has more • Counting – using number labels • 1:1 Correspondence • Cardinality - when counting, the last number gives the quantity • Hierarchical Inclusion– smaller numbers are part of bigger numbers; 1 more/less • Part / whole relationships • Compensation – thinking relationally $5+1=6$ so $4+2=6$ • Unitising– 1 unit can have a value of more than 1; $20=2\text{ tens}$; 2 groups of 10 or 1 group of 20 instead of 20 ones <p>Shumway, J. (2011)</p>	<ul style="list-style-type: none"> • Work with concrete materials and familiar ideas • Compose and recompose different arrangements and representations of number • Discuss and share their discoveries and solutions • Investigate the realistic uses of number in their everyday world • Explore number patterns and relationships • Create alternative methods of calculation and estimation • Solve realistic problems using a variety of approaches • Calculate for a purpose rather than just for the sake of calculating • Gather, organise, display and interpret quantitative data • Measure and estimate measure for a purpose • Explore very large numbers and their representations including using number lines <p>Jenni Back (2014)</p>

Addition

Aggregation

Combining two or more quantities,
E.g. Tom had two sweets and John had three sweets:
how many did they have altogether?

Augmentation

Enlarge something or to add to it.
E.g. Tom had two sweets and bought two more. How
many sweets does he have now?

Combine - Combination of 2 quantities.

E.g. *Mark has 3 books from the library. Susan has
borrowed 2 more books than Mark. How many books
does Susan have?*

Comparison: E.g. *119 boys took part in an art
competition. 15 more girls than boys took part. How
many girls took part in the competition?*

Equalise: E.g. *James has 30 books to read. If he gets 15
more, he will have the same amount as Chelsea. How
many does Chelsea have?*

Forms of Addition (Carpenter & Moser 1983)

Subtraction

Combine - Separation (into 2 quantities)

E.g. *James has eaten 5 healthy snacks and Charlie has
eaten 2. How many fewer healthy snacks has Charlie
eaten?*

Comparison: E.g. *134 girls took part in an art
competition. 15 fewer boys than girls took part. How
many boys took part in the competition?*

Equalise: E.g. *Philip had 65 stickers but lost 2 of them.
He now has the same as Jessie. How many does Jessie
have?*

Forms of Subtraction: (Carpenter & Moser 1983)

Multiplication

Whilst addition and subtraction are considered to involve joining sets, **multiplication is about replication.**

When teaching multiplication, it is important to move beyond the idea of repeated addition. While repeated addition can be a useful starting point, it can lead pupils to focus on only one number (the number being added repeatedly). This can limit their understanding. **Nunnes & Bryant (1996)**

Instead, multiplication should be taught as a relationship between two quantities: the number of groups and the number in each group. These two elements need to be thought about together. For example, in 4×6 , we are describing 4 groups of 6, not just adding 6 four times.

As teachers, we should use models such as arrays, bar models and grouping contexts to help pupils see both parts of the structure. This deeper understanding supports pupils in making sense of more complex concepts later, such as scaling, fractions of amounts and ratio.

Weak understanding (unary thinking):

Child sees: 4×6

Thinks: $6 + 6 + 6 + 6$

Focus = **6 only**

Strong understanding (binary thinking):

Child sees: 4×6

Thinks: 4 groups with 6 in each
or 6 groups with 4 in each.

Commutative Property

You can **switch the order** of the numbers, and the answer stays the same.
e.g. $5 \times 3 = 3 \times 5$

Associative Property

You can **change how numbers are grouped** when multiplying.
 $(5 \times 3) \times 2 = 5 \times (3 \times 2)$

Distributive Property

You can **break a number apart** to make multiplication easier.
 $5 \times (4 + 3) = (5 \times 4) + (5 \times 3)$

Compensation Property

You **adjust a number to make it easier**, then fix it.
e.g. $12 \times 9 =$
Think $12 \times 10 = 120$ which is easy to calculate
Now adjust by subtracting because we added one extra group of 12.
 $120 - 12 = 108$

Division

Sharing:

- In the sharing model we allocate or deal out items one at a time into each of the required groups. Considered to be division by the multiplier.
- Secures understanding of halving and one-to-one correspondence between objects
- Requires little knowledge or skill beyond counting
- Provides no image to support understanding of what to do with remainders
- Repeated subtraction of one
- The number of groups are known,
- Group size is not known
- *Also known as partitive division. E.g. Put the 12 into two groups. (Answer is how many in one group)*

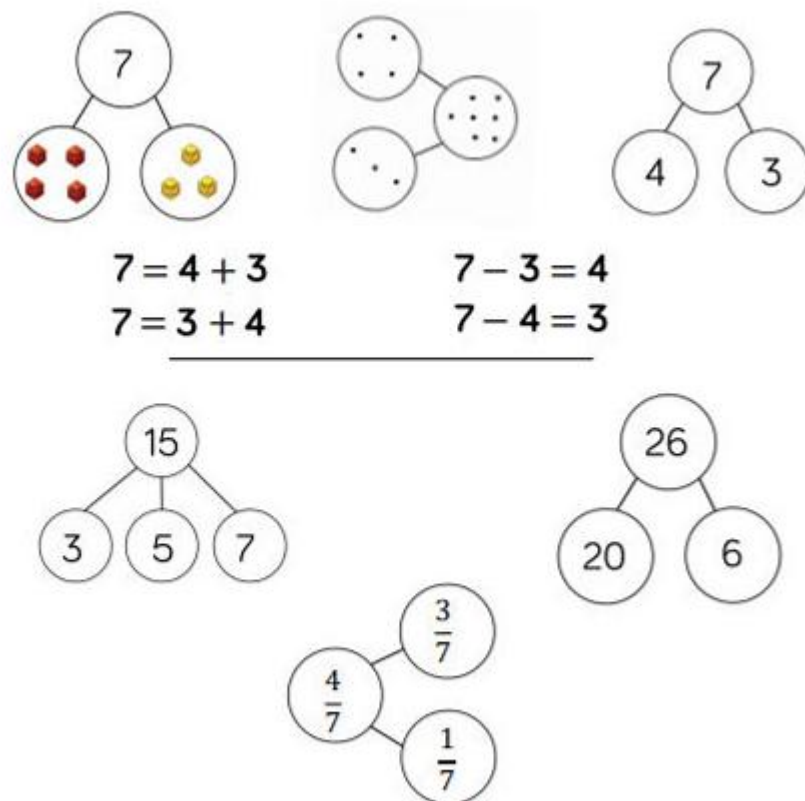
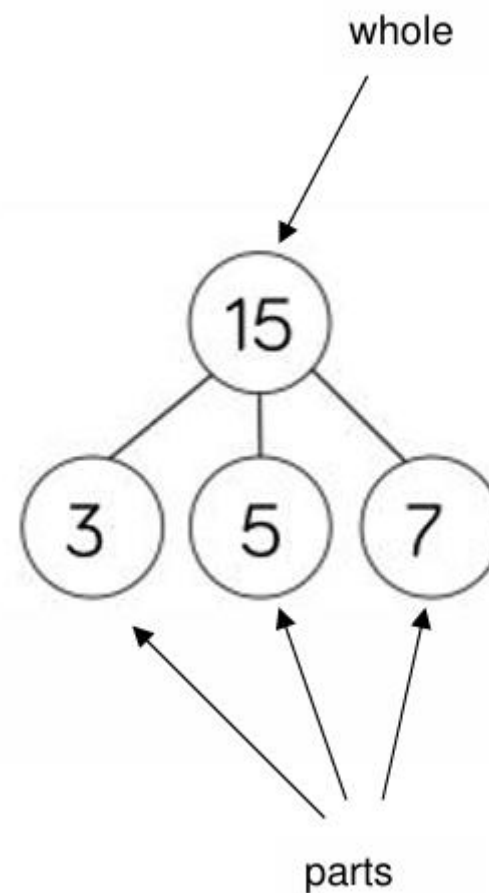
Grouping:

- In the grouping structure you gather groups of objects at a time. This is associated with repeated subtraction, but it is naturally treated as repeated addition. How many 2's in 24? (*Identify 3 ways*)
- Secures understanding that the divisor is important in the calculation
- Requires sound knowledge of addition and subtraction facts
- Provides an image to support understanding of what to do with remainders
- Provides a firmer basis on which to build pupils' understanding of division
- Repeated subtraction of a group
- Group size is known
- Number of groups is not known
- *Also known as quotative division E.g. Put the 12 into groups of 2. (Answer is the number of groups)*

Representations

Part-Whole Model

Part-Whole models can support children as they learn to partition numbers. As children move into the Upper Prep, they can be used to partition fractions, percentages and decimals. They are a useful diagram to help children solve missing number calculations (when a part is missing) or understand that the parts create the whole by adding.



Bar Model

Similar to a part-whole model, a bar model can help children identify missing numbers by subtracting or calculate the whole by adding.



2,394

1,014

1,380

$$2,394 - 1,014 = 1,380$$

They can help children turn a calculation into something visual

E.g $432 + ? = 790$

790	
432	?

40									
4	4	4	4	4	4	4	4	4	4

Bar models are a really useful way to explain division when the bar is split equally or work out fraction of amounts e.g. $\frac{3}{10}$ of 40.

Tim and Sally share marbles in the ratio of 2:3

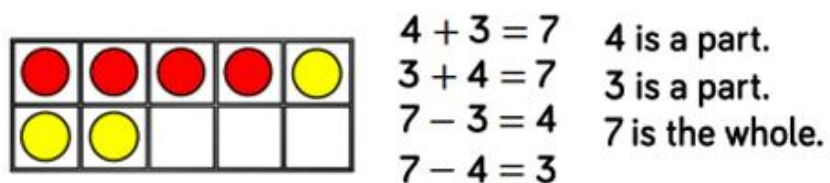
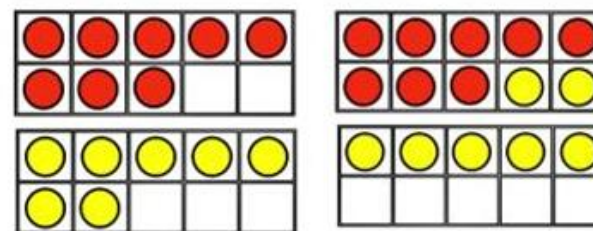
If Sally has 36 marbles, how many are there altogether?



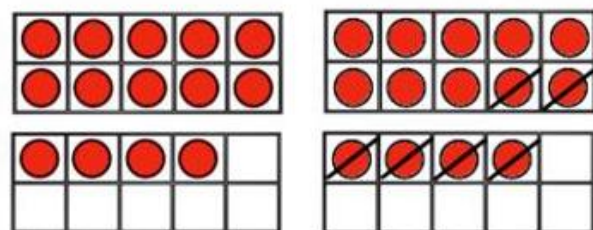
Ten Frames (and Five Frames)

Ten frames and five frames can be used to help children add and subtract, they can allow children a hands-on experience of increasing or decreasing the number of counters. This complements the teaching of part-whole models where children understand that the whole can be split into parts and the parts can be totalled to create the whole. Counters do not always need to be used and children can draw different coloured circles to represent the counters.

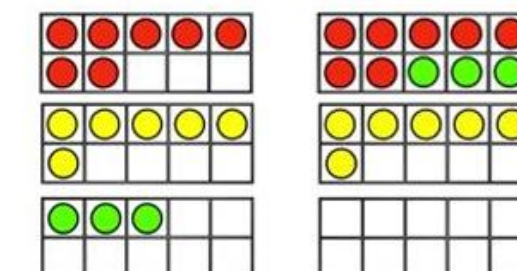
As children become more number confident, they will be able to apply this knowledge to ten frames within 20. By identifying number bonds, children will be able to build on their mental calculations.



$$\begin{array}{r} 8 + 7 = 15 \\ \swarrow \quad \searrow \\ 2 \quad 5 \end{array}$$

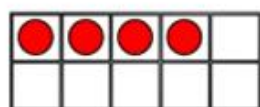


$$\begin{array}{r} 14 - 6 = 8 \\ \swarrow \quad \searrow \\ 4 \quad 2 \end{array}$$

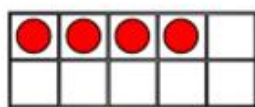


$$\begin{array}{r} 7 + 6 + 3 = 16 \\ \swarrow \quad \searrow \\ 10 \end{array}$$

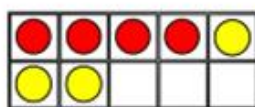
First



Then

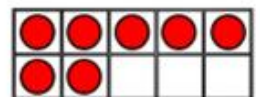


Now

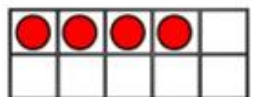


$$4 + 3 = 7$$

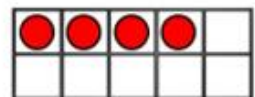
First



Then



Now



$$7 - 3 = 4$$

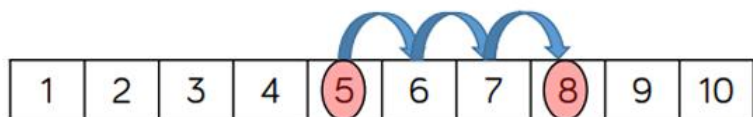
Number Lines

To begin with, children may recognise these as 'number tracks'. Number tracks support the introduction of a number line and the concept of jumping along a line to solve calculations. They can be used to help children solve addition and subtraction calculations.

Number Lines support children with adding and subtracting and can be used alongside a hundred square to mirror the jumps. Using a number line is a useful method to count on or back. As they grow in confidence, their jumps may be greater than ones. Children can use a physical number line, or they may draw one. It is really important that each jump touches the interval line which is labelled. Missing this out can cause mistakes and confusion.

When children are confident enough, the set intervals on the number line can be removed and children will start to use a blank number line and complete the labels underneath for themselves. This allows them to make more efficient jumps and use their understanding of partitioning and bonds to 10. The number lines can continue to be used for addition and subtraction; either using it to count on or back.

$$5 + 3 = 8$$



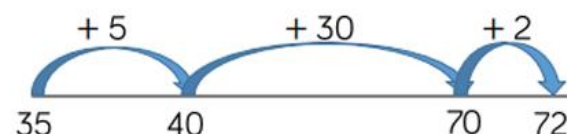
$$5 + 3 = 8$$



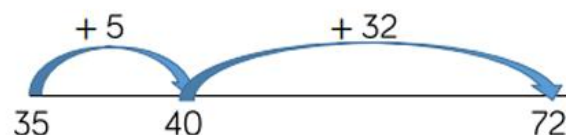
What number is halfway between 10 and 20?



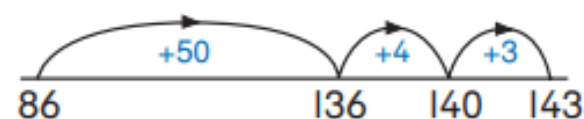
$$35 + 37 = 72$$



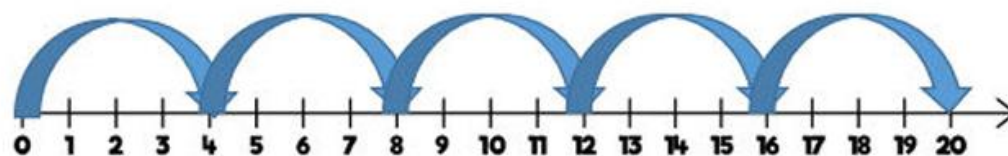
$$35 + 37 = 72$$



$$86 + 57 = 86 + 50 + 7 = 136 + 7 = 143$$

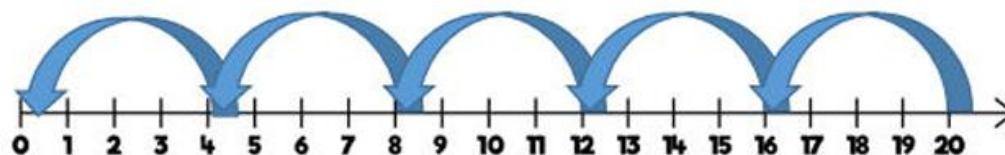


Number lines can also be very effective when teaching children to multiply and divide. Again, it is really important that the children recognise the need for accurate jumps which land on the interval line and it is labelled (if using a blank number line to start with). It is important that children understand that multiplication requires them to count on, whereas division will require them to count back. As children move through Upper Prep, they can use number lines for more challenging multiplication and division calculations.



$$4 \times 5 = 20$$

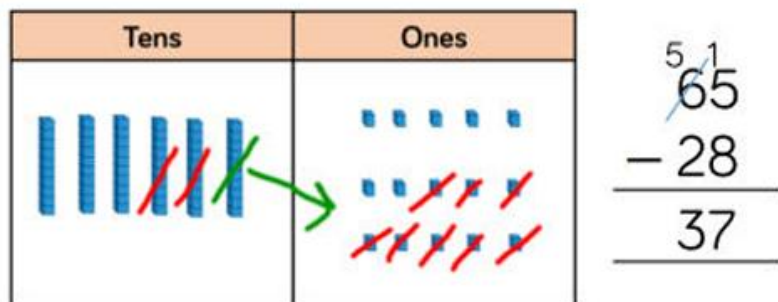
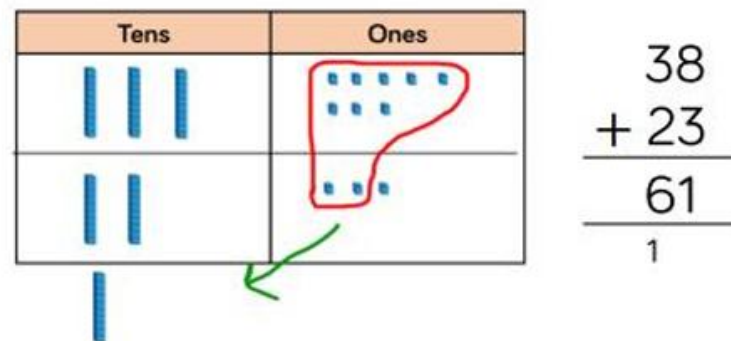
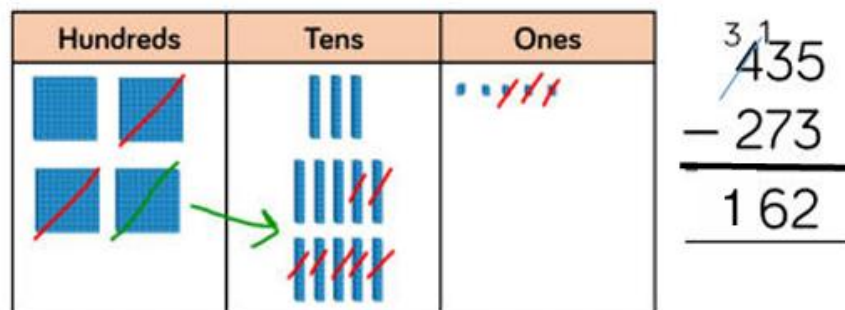
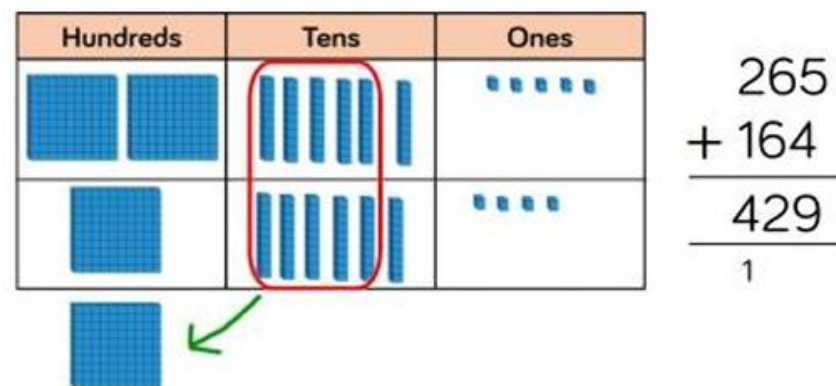
$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

Base 10 or Dienes

Base 10 or Dienes can be very useful to help children understand place value. Place value is essential to help children grasp the concept of numbers. The children come to recognise the value of each resource: ones, tens, hundreds and thousands. These resources can be used to support children with the learning of addition and subtraction.

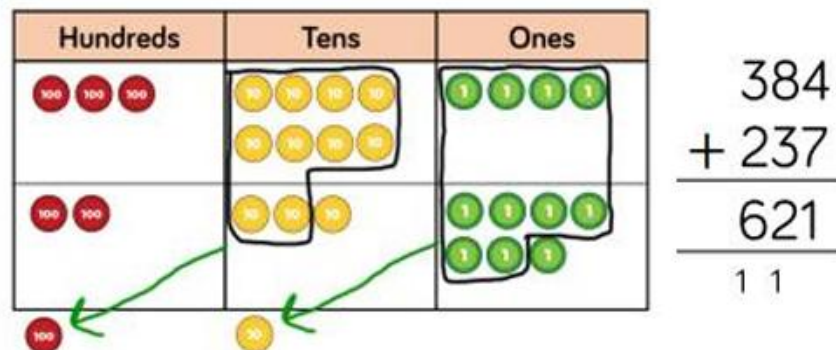
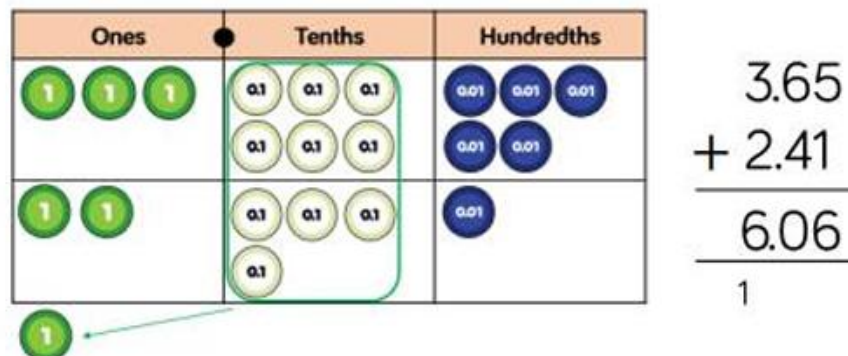
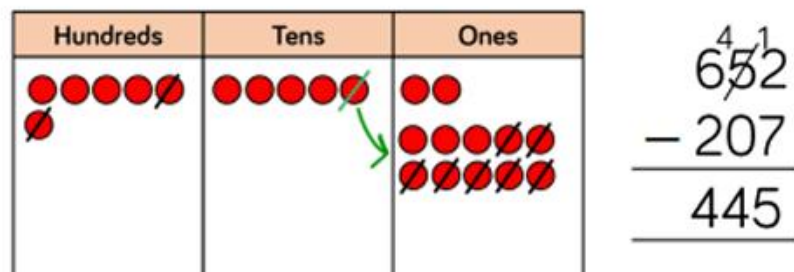
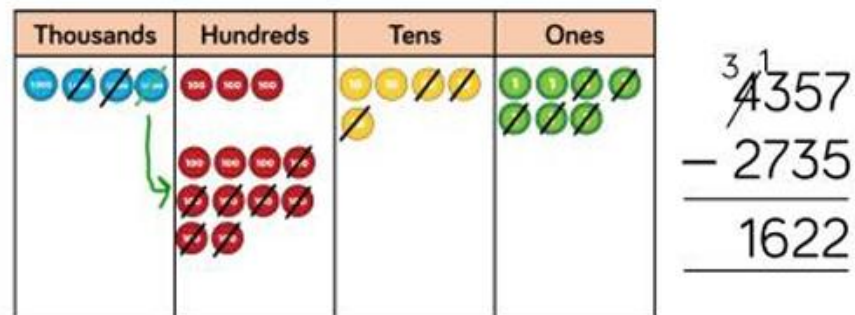


NB It is important to fully explain the transition between CPA. Children need to become the 'expert' in how to transfer from one model to the other.

'The practice of concreteness fading is effective in helping learners move from an understanding based on concrete objects to a more abstract understanding where they can solve problems without using the concrete objects.' (Fulmer 2017)

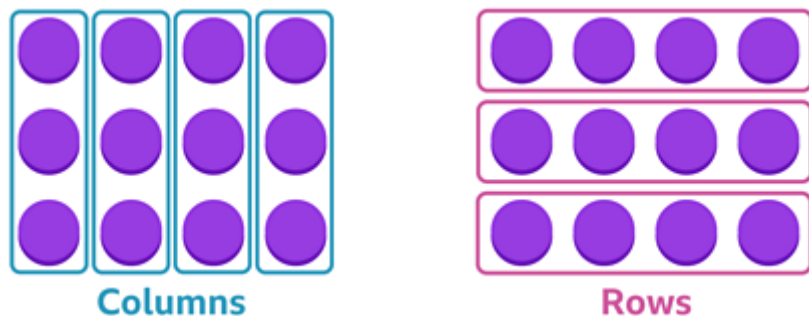
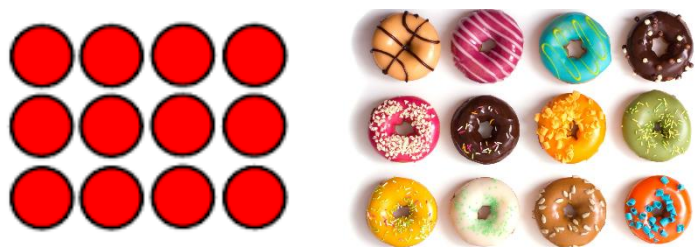
Place Value Counters

In a similar way, place value counters can also be used to support the children's understanding of place value, adding and subtracting.



Array

Children may also use an array when learning to multiply, displaying the concrete resources alongside the written method can be helpful for children to make the connection. Similarly, they could use the counters when dividing to share into equal groups.



An **array** in mathematics is a set of objects, pictures, or numbers arranged in equal rows and columns. It is a visual tool used to make counting, multiplication, and division easier to understand.

E.g. The product of 4 and 3 is 12. Three, four times is equal to 12. 3 and 4 are factors of 12. 3 times 4 is equal to 12.

Written Methods

It is vital that children have a consistent approach when learning written methods. Altering language or placement of digits can cause children great difficulty. Where year groups do not introduce a new written method, children will be consolidating their learning and developing the method by applying it in different contexts. It should be noted, that in Lower Prep, Reception, Year 1 and Year 2 will use a range of methods such as part-whole models, number lines, bar models and practical resources to teach the four operations, as well as recording calculations appropriate to their ability. Number writing is also practised regularly in Reception and Year 1. There will also be a focus on practical maths.

The Concrete-Pictorial-Abstract (CPA) Approach

Key Aspects of the CPA Approach

- **Concrete ("Doing"):** Learners use physical objects—manipulatives like cubes, counters, Dienes blocks, or fruit—to model a mathematical concept. This brings abstract ideas to life through hands-on experience.
- **Pictorial ("Seeing"):** Pupils represent the physical objects through drawings, diagrams, or charts, such as bar models, number lines, or sketches. This stage bridges the gap between physical reality and symbolic representation.
- **Abstract ("Symbolic"):** Students use formal symbols, numbers, and signs (E.g. $+$, $-$, $\times$, $\div$) to represent the problem, demonstrating a secure understanding of the underlying concepts.

Core Principles and Benefits

- **Not Strictly Linear:** While taught in order, learners can move back and forth between stages to solidify understanding (e.g., returning to cubes to check an abstract calculation).
- **Improves Understanding:** By delaying the use of abstract symbols, students understand *what* the math means, not just how to get the answer.
- **Reduces Anxiety:** Provides a supportive, step-by-step framework that builds confidence and reduces anxiety.
- **Applicable for All Ages:** While often associated with primary math, CPA is effective for complex topics in higher years.

Implementing CPA

- **Consistent Representation:** Use consistent pictorial models across year groups to build familiarity.
- **Create 'experts':** Providing opportunities for staff and pupils to understand the functionality of the manipulative both in limitation and application across numerous mathematical concepts.
- **Create 'Stop' points in the curriculum:** Times when the manipulative itself needed to be taught to the pupil regardless of the programme of study, it was deemed necessary to provide the pupil with expertise in the 'tool' (*Jenni Back 2013*) before applying it to the concept being taught for that year group.

- **Transition/Transfer points:** Ensure the pupils understand how the manipulatives concreteness may need to be adapted to ensure transfer occurred effectively, avoiding misconception from C to P to A. It is important to provide pupil with the skill set to practice and understand the transition between Concrete, Pictorial (Representational) and Abstract is essential/critical for success to be obtained.

Considerations during Application

How can I use manipulatives effectively in my teaching?

- As educators, we need to carefully consider the manipulative, our learners, and the pedagogy to ensure that pupils' work with manipulatives supports their learning.
- Select the right manipulative for the task and your students.
- Structure the environment for effective learning.
- Support transfer from concrete to abstract.

Select the Right Manipulative for the Task and your pupils.

- What do you want pupils to know or be able to do by the end of the lesson?
- What is the role or purpose of the manipulative within the lesson?
- What features must the manipulative have to support learning of the target concept?
- What features of the manipulative might distract learners or lead them astray?
- What prior knowledge or experience might pupils have with this manipulative, and how could that impact their learning?

Structure the Environment for Effective Learning.

- How will I communicate the purpose of the manipulative to pupils?
- How will I help students focus their attention on the relevant features of the manipulative?
- What prior knowledge do pupils need to use the manipulative effectively to understand the concept?
- How will I model and scaffold use of the manipulative for students?

Models to Support Calculation

Rule: modelling/explanations must be built upon prior knowledge for success to be obtained. This is when new knowledge is connected to existing knowledge. (*Thomas Wolsey stated, 'Be very, very careful what you put into that head because you will never get it out.'*)

Rule: Effectiveness of modeling/explaining is limited by the capacity of working memory. To learn we must transfer information from working memory to long-term memory so it can be stored and later retrieved.

Rule: The single most important rule is to accommodate the limitations of working memory; we need to regulate the amount of cognitive work/load demanded during modeling/explanation and practice. *John Sweller (2006)*.

Rule: Cognitive bias relates to how the novice learner can lack the understanding required to be able to correctly assess whether they understand the explanation/modeling or not. (They may think they understand but are unable to accurately make that judgement independently. We ask questions for two reasons:

Worked Examples

Worked Example Pairs	Incomplete Worked Examples	Correct and Incorrect Worked Examples	Strategy Comparison worked Examples
<ul style="list-style-type: none">• Teacher models example, pupils complete theirs.• Pupil question is minimally different.• Withhold pupil question until worked example is completed.• Get pupils to answer on mini whiteboards as check for understanding.• Respond accordingly• Use earlier in the learning episode.	<ul style="list-style-type: none">• Scaffold practice (including retrieval practice).• One or more missing step.• Gradual reduction/ fading more effective than. transferring from worked examples to un-scaffolded problems abruptly.• Backwards fading (missing the last step) is the simplest form.• Use early in the learning episode.	<ul style="list-style-type: none">• Allows for direct comparison – Correct v Incorrect.• Expose misconceptions.• No more than one misconception per worked example.• Use later in the learning episode.• Perfect for assessment.• Most effective when paired with pupil self-explanation prompts.	<ul style="list-style-type: none">• Same problem solved in different ways.• Deepen conceptual understanding.• Identifies similarities and differences• Make references to prior knowledge.• Promotes variation.• Provides opportunities to develop structures and representations.• Encourages thinking mathematically.

Goal Free Problems (Sweller)

Problem solving is the key of 'doing' and also learning to do mathematics and takes the fundamental role of developing mathematical knowledge.

What do you see?

What do you know?

What can you tell me?

What else could it be?

A flexible thinker includes the ability to apply knowledge in different contexts including the ability to apply skills to unfamiliar situations rather than simply use it in similar context when it is studied.

Cognitive Load Theory suggests that by reducing extraneous cognitive load during learning could enhance the transference of learning from one context to another.

A goal free problem strategy that is developed based in cognitive load theory have been showed to be effective for transfer learning. The instruction in a goal-free problem directs pupils to 'calculate as many solutions as you can' rather than to calculate a single given goal.

Solving unfamiliar problems can place a strain on working memory, particularly for the 'novice' learner (those with less knowledge and experience of solving problems) who tend to solve problems in an inefficient way.

The complication for the novice learner is that most problems have a goal or endpoint, therefore when faced with an unfamiliar problem pupils tend to work backwards from the goal.

Isolate the Skill

According to Daisy Christodoulou 2017, *'If we want pupils to develop a certain skill, we have to break that skill down into its component parts and help pupils to acquire the underlying mental model.'*

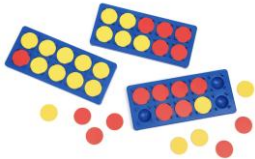
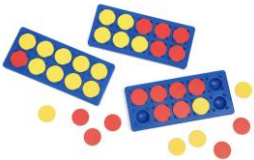
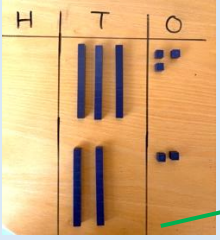
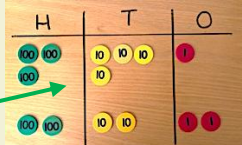
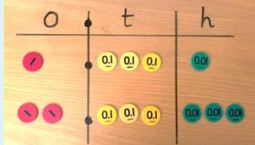
Rather than presenting problems in their entirety continually, which can lead to cognitive overload, the problem should be broken down into smaller components, crafted in a carefully chosen order, that accumulate to achieve greater success with a larger, well-defined outcome.

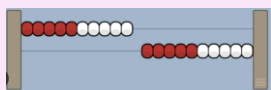
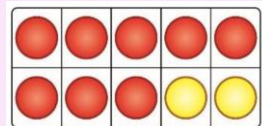
Analogy can be made in Art. A painter needs the skill of all the component parts in order to produce a painting (*drawing, perspective, colour use, paint technique, brush manipulation etc*).

Atomisation (*Bruno Reddy blog*) is when a problem is broken down into small concise steps. The teacher then needs to identify which steps are known to the pupil (check with practice) and which need to be taught to complete the problem in its entirety.

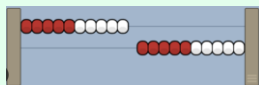
This process is best identified before any teaching block so the route to achieving the goal is made explicit to the pupils. Lessons become much easier and quicker to plan as teachers can simply work through each step identified.

Breckon Hill Primary School Calculation Policy (Addition)

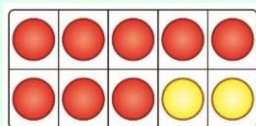
	Early Years	Year One	Year Two	Year Three	Year Four	Year Five	Year Six
Concrete	<p>Children use rekinreks to recognise how to add numbers to find the total efficiently</p>  <p>Use five frames to represent key number bonds to five and ten.</p>  <p>8 and 2 is 10. There are 10 altogether.</p>	<p>Children use rekinreks to recognise how to add numbers to find the total efficiently</p>  <p>Use ten frames and part-whole models to represent key number bonds.</p>  <p>8 and 2 is 10. There are 10 altogether.</p>	<p>Add the 10s and 1s separately.</p>  <p> $3 + 2 = 5$ There are 5 ones in total. $3 \text{ tens} + 2 \text{ tens} = 5 \text{ tens}$. There are 5 tens in total. $33 + 22 = 55$ </p>	<p>Ensure columns are added from right to left. Remember to include any exchanges in the calculation.</p> 	<p>Continue to use place value counters in Year 4</p>	<p>Continue to use place value counters in Year 5. With decimals too.</p> 	<p>Continue to use place value counters in Year 6</p>



Children use pictorial representations of rekinreks to recognise how to add numbers to find the total efficiently

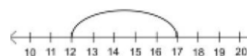


Use ten frames and part-whole models to represent key number bonds.



Start at the larger number on the number line and count on in 1s to find the answer.

$$12 + 5 = 17$$



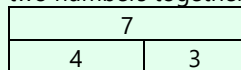
Use part-whole model to add two numbers together.



$$7 = 4 + 3$$

$$7 = 3 + 4$$

Use a bar model to add two numbers together



$$4 + 3 = 7$$

$$3 + 4 = 7$$

Add the 10s and 1s separately.



$$5 + 3 = 8$$

There are 8 ones in total.

$$3 + 2 = 5$$

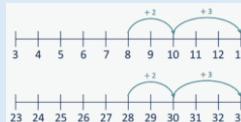
There are 5 tens in total.

$$35 + 23 = 58$$

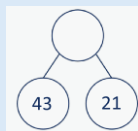
Pupils' pictorial representation of 48 may look more like this:



Use a number line to add in greater amounts than 1

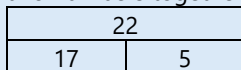


Add two 2 digit numbers using a part whole model

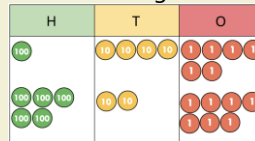


$$43 + 21 = 54$$

Use a bar model to add two numbers together



Ensure columns are added from right to left.

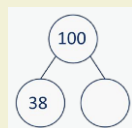
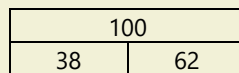


Remember to include any exchanges in the calculation.

Continue to use number lines to support the addition

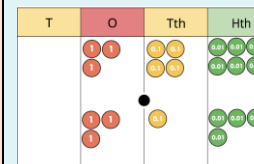


Transition from part-whole to bar model by showing representations alongside each other. Eventually, part-whole will be removed.



Continue to use place value counters in Year 4

Continue to use place value counters in Year 5. Decimals addition.



Continue to use place value counters in Year 6

3	+	4	=	7
---	---	---	---	---

3	+	4	=	7
---	---	---	---	---

1	3	+	1	4	=		
1	0	+	1	0	=	2	0
3	+	4	=	7			
2	0	+	7	=	2	7	

Add the 1s. Then add the 10s.

		T	O	
		3	2	
	+	1	4	
			6	

		T	O	
		3	2	
	+	1	4	
		4	6	

Add the 1s. Exchange 10 ones for a ten. Then add the 10s.

		T	O	
		3	6	
	+	2	9	
			5	
		1		

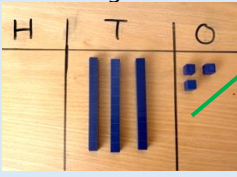
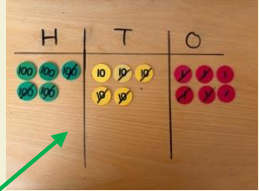
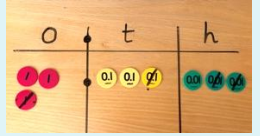
		T	O	
		3	6	
	+	2	9	
		6	6	
		1		

	H	T	O	
	2	6	5	
+	1	6	4	
	4	2	9	
	1			

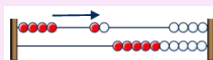
	TH	H	T	O
	1	3	7	8
+	2	1	4	8
	3	5	2	6
		1	1	

	T	O	t	h
		3	6	5
+		2	4	1
		6	0	6
		1		

Breckon Hill Primary School Calculation Policy (Subtraction)

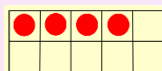
	Early Years	Year One	Year Two	Year Three	Year Four	Year Five	Year Six
Concrete	<p>Use real life objects to support understanding of number. E.g. Magnitude, counting.</p>  <p>Number sentences. 1 less than ... is ...</p>  <p>$6 - 2 = 4$</p>	<p>Represent and use number bonds and related subtraction facts within 20. Subtract one-digit and two-digit numbers to 20, including zero. $6 - 2 = 4$</p> 	<p>Subtract numbers using concrete objects</p> <ul style="list-style-type: none"> • a two-digit number and 1s • a two-digit number and 10s • 2 two-digit numbers.  <p>$33 - 21 =$</p>	<p>Subtract numbers with up to three digits</p>  <p>$556 - 345 =$</p>		<p>Subtract decimals with up to 2 decimal places</p> 	

Number sentences.
1 less than ... is ...

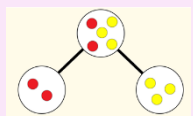


$$6 - 2 = 4$$

Used to support early calculation within 10.



$$4 - 3 = ?$$

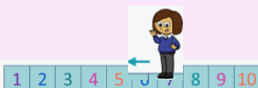


$$5 - 3 = ?$$

$$5 - 2 = ?$$

= ?

Count back from any number within 10



Represent and use number bonds and related subtraction facts within 20. Subtract one-digit and two-digit numbers to 20, including zero.



$$6 - 2 = 4$$

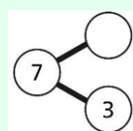
Used to support early calculation within 20.



$$5 - 4 =$$

Children use a part-whole model to support the subtraction to find a missing part.

$$7 - 3 = ?$$

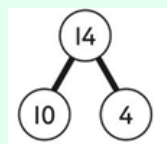


$$19 -$$

$$14$$

$$19 -$$

$$10 =$$



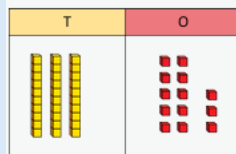
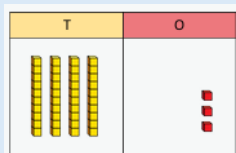
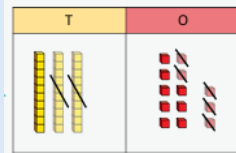
$$9$$

$$9 - 4 = 5$$

$$\text{So, } 19 - 14 = 5$$

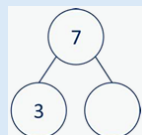
Subtract numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and 1s
- a two-digit number and 10s
- 2 two-digit numbers.



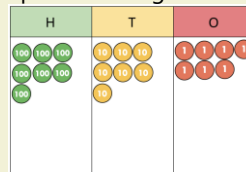
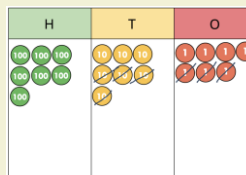
$$43 - 25 = ?$$

If ... is a whole and ... is a part, then ... is the other part.

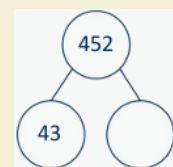


$$7 - 3 = ?$$

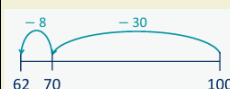
Subtract numbers with up to three digits



$$777 - 43 = ?$$

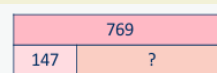


$$452 - 43 =$$

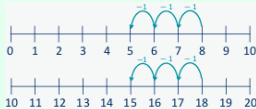
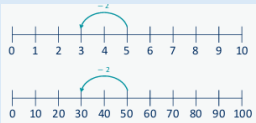


$$100 - 38 =$$

Use bar model so solve subtraction.

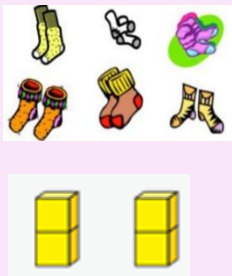
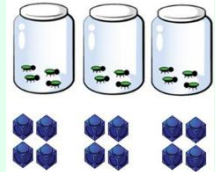
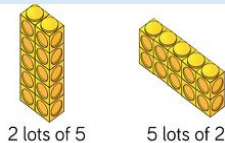
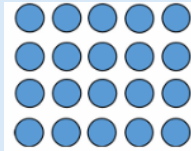
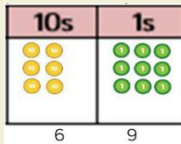
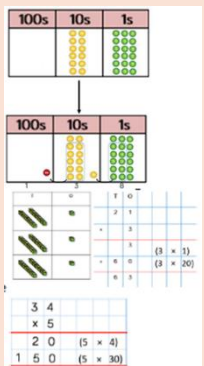


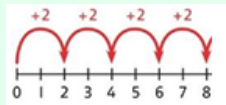
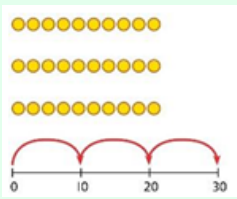
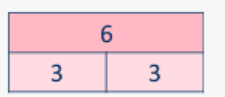
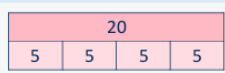
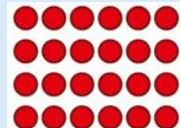
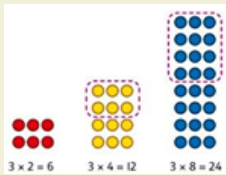
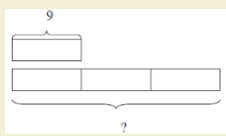
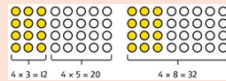
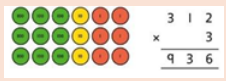
$$769 - 147 =$$

		<p>Subtract numbers within 20.</p> 	<p>Recall and use subtraction facts to 20 fluently and derive and use related facts up to 100.</p> 																																																																																																																																																					
Abstract	<p>Number sentences. 1 less than ... is ... $4 - 3 = ?$</p>	<p>Children understand 'find the difference' as subtraction.</p> <p>$10 - 4 = 6$ The difference between 10 and 6 is 4.</p> <p>Understand how to use knowledge of bonds within 10 to subtract efficiently. $5 - 3 = 2$ $15 - 3 = 12$</p>	<p>Using column subtraction to exchange.</p> <table data-bbox="770 533 927 708"><tr><td></td><td>T</td><td>O</td></tr><tr><td></td><td>3</td><td>15</td></tr><tr><td>-</td><td>2</td><td>7</td></tr><tr><td></td><td>1</td><td>8</td></tr><tr><td></td><td></td><td></td></tr></table> <p>Subtraction facts to 20 and derive and use related facts up to 100. $15 - 6 = ?$ $35 - 6 = ?$</p>		T	O		3	15	-	2	7		1	8				<p>Use formal algorithms to solve subtraction calculations.</p> <table data-bbox="1061 604 1263 780"><tr><td></td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td>3</td><td>4</td><td>15</td></tr><tr><td>-</td><td>2</td><td>7</td><td>3</td></tr><tr><td></td><td>1</td><td>6</td><td>2</td></tr><tr><td></td><td></td><td></td><td></td></tr></table>		H	T	O		3	4	15	-	2	7	3		1	6	2					<p>Use formal algorithms to solve subtraction calculations.</p> <table data-bbox="1341 604 1588 780"><tr><td></td><td>TH</td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td>3</td><td>4</td><td>13</td><td>5</td></tr><tr><td>-</td><td>2</td><td>7</td><td>3</td><td>5</td></tr><tr><td></td><td>1</td><td>6</td><td>2</td><td>2</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></table>		TH	H	T	O		3	4	13	5	-	2	7	3	5		1	6	2	2						<p>Use formal algorithms to solve subtraction calculations.</p> <table data-bbox="1610 604 1868 780"><tr><td></td><td>HTH</td><td>TTH</td><td>TH</td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td>2</td><td>9</td><td>9</td><td>3</td><td>8</td><td>2</td></tr><tr><td>-</td><td>1</td><td>8</td><td>8</td><td>5</td><td>0</td><td>1</td></tr><tr><td></td><td>1</td><td>1</td><td>1</td><td>8</td><td>8</td><td>1</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table> <table data-bbox="1610 820 1868 995"><tr><td></td><td>T</td><td>O</td><td>t</td><td>h</td></tr><tr><td></td><td></td><td>4</td><td>5</td><td>14</td></tr><tr><td>-</td><td></td><td>2</td><td>7</td><td>0</td></tr><tr><td></td><td></td><td>2</td><td>7</td><td>3</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></table> <p>$5.43 - 2.70 =$</p>		HTH	TTH	TH	H	T	O		2	9	9	3	8	2	-	1	8	8	5	0	1		1	1	1	8	8	1									T	O	t	h			4	5	14	-		2	7	0			2	7	3						<p>Use formal algorithms to solve subtraction calculations.</p> <table data-bbox="1890 604 2136 780"><tr><td></td><td>T</td><td>O</td><td>t</td><td>h</td></tr><tr><td></td><td></td><td>6</td><td>6</td><td>13</td></tr><tr><td>-</td><td></td><td>1</td><td>3</td><td>4</td></tr><tr><td></td><td></td><td>5</td><td>3</td><td>9</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></table>		T	O	t	h			6	6	13	-		1	3	4			5	3	9					
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Breckon Hill Primary School Calculation Policy (Multiplication)

- Concrete, Pictorial and abstract approach should be taught simultaneously whenever a new concept is introduced, revisited or when the teacher wishes to build further on the concept. Pupils need to see the concrete, pictorial (bridge between C and A) and abstract completed concurrently to secure links, differences in procedure and transition between each one. CPA should not be taught in isolation.
- Pupils who are not secure in their table's facts will find multiplication calculations difficult to complete. Where this is the case, it is advised that teachers return to the principle of multiplication and focus on table fact automaticity, number facts, addition and subtraction and rules of multiplication.
- E.g. 26×3 is $26 + 26 + 26$.
- Counting Stick, Number Line, Arrays and Cubes are excellent models to gain an understanding of tables facts, how they relate to each other and provide context for the core principles of multiplication - commutative, distributive, Associative and Distributive.
- Bar model provides a visual representation to support pupils in calculating multiplication tasks, it must be noted that this model depends on pupils having a secure knowledge of tables facts and the principle of multiplication.
- Base Ten, Place Value counters provide concrete (doing stage – enactive) opportunities for pupils to understand the principle of multiplication. It must be noted as calculations become more complicated, a concrete approach will increase cognitive load on a pupil and create misconceptions.

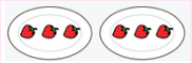
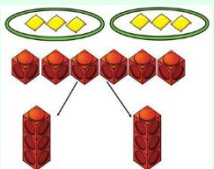
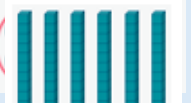
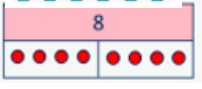
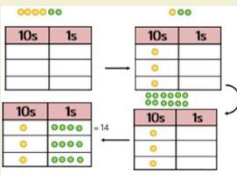
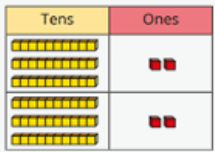
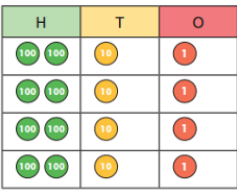
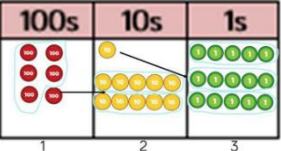
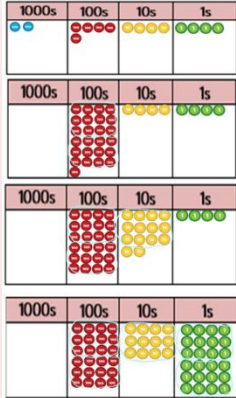
	EY	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Concrete	<p>Experiencing equal groups of objects. Instruction of doubling.</p> 	<p>Counting tables facts (multiples), recognise patterns and recall.</p> <p>Repeated</p>  <p>grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.</p> 	<p>Number lines to show repeated groups- 3×4.</p>  <p>Use arrays to illustrate commutativity counters and other objects can also be used.</p> <p>$2 \times 5 = 5 \times 2$ $2+2+2+2+2=10$ $5+5=10$</p>  <p>2 lots of 5 5 lots of 2</p> <p>$5+5+5+5+5=20$ $4 \times 5 = 20$ $5 \times 4 = 20$</p> 	<p>Formal column method with place value counters (base 10 can also be used.) 3×23</p> <p>Explore the</p>  <p>relationship between known times-tables and multiples of 10 using place value equipment. Make 4 groups of 3 ones.</p>  <p>Make 4 groups of 3 tens.</p>  <p>What is the same? What is different?</p>	<p>Formal column method with place value counters. 6×23</p> <p><i>NB Use with caution. Pupils need to be secure with the principles of multiplication to attempt this model.</i></p>  <p>Use place value equipment to make multiplication.</p> <p><i>E.g. Make 4×136</i> I can work out how many 1s, 10s and 100s. There are 4×6 ones... 24 ones There are 4×3 tens ... 12 tens There are 4×1 hundreds ... 4 hundreds $24 + 120 + 400 = 544$</p>  <p>$+ 400 = 544$</p>	<p>When children start to multiply 3 digits $\times 3$ digits and 4 digits $\times 2$ digits etc., they should be confident with the abstract: To get 744 children have solved 6×124. To get 2480 they have solved 20×124.</p>	

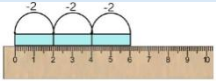
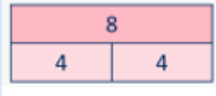
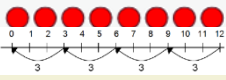
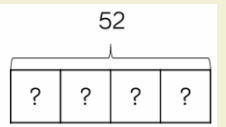
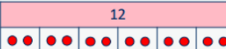
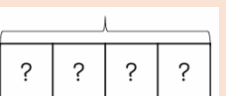
Pictorial	<p>There are ... groups of... There are ... altogether.</p> 	<p>Counting tables facts (multiples), recognise patterns and recall.</p>  <p>$4 \times 2 = 8$</p>  <p>Understand how to relate counting in unitised groups and repeated addition with</p>  <p>knowing key times-table facts. $10 + 10 + 10 = 30$ $3 \times 10 = 30$</p>	<p>Number lines to show repeated groups- 3×4.</p>   <p>Bar Model $2 \times 3 = 6$ $3 + 3 = 6$</p>  <p>$4 \times 5 = 20$ $5 + 5 + 5 + 5 = 20$</p> <p>Use arrays to illustrate commutativity counters and other objects can also be used.</p> <p>$6 \times 4 = 24$</p>  <p>$4 \times 6 = 24$ $3 \times 4 + 2 \times 4 = 24$</p>	<p>Children recognise that arrays demonstrate commutativity.</p>  <p>This is 3 groups of 4. This is 4 groups of 3.</p> <p>Children understand how the $\times 2$, $\times 4$ and $\times 8$ tables are related through repeated doubling.</p>  <p>Bar Model E.g. $9 \times 3 =$</p> 	<p>Understand how multiplication and partitioning are related through addition.</p>  <p>$4 \times 3 = 12$ $4 \times 5 = 20$ $12 + 20 = 32$ $4 \times 8 = 32$</p> <p>Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.</p> 	<p>Bar Model Writtn Problems E.g. Peter has 4 books. Harry has five times as many books as Peter. How many books has Harry?</p>	
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Abstract		<p>Explore the relationship between multiplication and repeated addition.</p> <p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>	<p>Children to be able to use an array to write a range of calculations</p> <p>E.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$</p>	<p>Children to record what it is they are doing to show understanding.</p> <p>E.g. $23 \times 3 =$ $3 \times 20 = 60$ $3 \times 3 = 9$ $60 + 9 = 69$</p> <div>$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$</div>	<p>Formal written method.</p> <p>$6 \times 23 =$</p> <div>$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ 11 \\ \hline \end{array}$</div>	<p>When children start to multiply 3 digits $\times$ 3 digits and 4 digits $\times$ 2 digits etc., they should be confident with the abstract model.</p> <div>$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ 11 \\ \hline \end{array}$<p>Answer: 3224</p></div>	<p>Use this model to identify incremental facts.</p> <table><tr><th>$\times 32$</th><th></th></tr><tr><td>1</td><td>32</td></tr><tr><td>2</td><td>64</td></tr><tr><td>5</td><td>160</td></tr><tr><td>10</td><td>320</td></tr><tr><td>20</td><td>640</td></tr><tr><td>50</td><td>1600</td></tr><tr><td>100</td><td>3200</td></tr></table>	$\times 32$		1	32	2	64	5	160	10	320	20	640	50	1600	100	3200
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Breckon Hill Primary School Calculation Policy (Division)

- Concrete, Pictorial and abstract approach should be taught simultaneously whenever a new concept is introduced, revisited or when the teacher wishes to build further on the concept. Pupils need to see the concrete, pictorial (bridge between C and A) and abstract completed concurrently to secure links, differences in procedure and transition between each one. CPA should not be taught in isolation.
- Pupils who are not secure in their table's facts will find division calculations difficult to complete. Where this is the case, it is advised that teachers return to the principle of multiplication and focus on table fact automaticity, number facts and rules of division. E.g. $26 \div 2$ is 13 groups of 2 or 26 shared equally between 2.
- Counting Stick, Number Line, Arrays and Cubes are excellent models to gain an understanding of tables facts, how they relate to each other and provide context for the core principles of division - grouping, sharing.
- Bar model provides a visual representation to support pupils in calculating multiplication tasks, it must be noted that this model depends on pupils having a secure knowledge of tables facts and the principle of division.
- Base Ten, Place Value counters provide concrete (doing stage – enactive) opportunities for pupils to understand the principle of division. It must be noted as calculations become more complicated, a concrete approach will increase cognitive load on a pupil and create misconceptions.

	EY	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Concrete	<p>Sharing Provide practical activities. Pupils to check whether items have been shared fairly (equally).</p> <p>Grouping Provide opportunities to make equal groups.</p> <p>Pupils to check that each group has the same amount.</p>  	<p>Skip counting backwards, find equal amounts.</p>  <p>Sharing using a range of objects. E.g. $6 \div 2$</p> 	<p>Recall and use division facts for the 2, 5 and 10 multiplication tables. $4 \times 2 = 8$ $8 \div 2 = 4$</p>   <p>There are 6 equal groups of 10. $6 \times 10 = 60$ $60 \div 10 = 6$</p>	<p>Sharing using place value counters. E.g. $42 \div 3 = 14$</p>   <p>$60 \div 2 = 30$ $4 \div 2 = 2$ $64 \div 2 = 32$</p>	<p>Short division using place value counters to group. E.g. $844 \div 4$</p>  <p>E.g. $96 \div 4$</p>  <p>NB Exchange a 10 for 10 ones $80 \div 4 = 20$ $16 \div 4 = 4$ $96 \div 4 = 24$</p>	<p>Short division using place value counters to group. E.g. $615 \div 5$</p>  <p>1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?</p>	<p>Long division using place value counters. E.g. $2544 \div 12$</p> 

Pictorial		<p>Solve simple one-step problems involving division using arrays. There are 12 counters altogether. How many groups</p>  <p>can you make?</p>	<p>Repeated subtraction. E.g. $6 \div 2$</p>  <p>3 groups of 2</p> <p>Recall and use division facts for the 2, 5 and 10 multiplication tables. $4 \times 2 = 8$ $8 \div 2 = 4$</p>  	<p>Recall and use division facts for the 3, 4 and 8 multiplication tables. Write and calculate mathematical statements for division using the multiplication tables that they know, including for 2-digit numbers times 1-digit numbers, using mental and progressing to formal written methods. $2 \times 8 = 16$ $16 \div 8 = 2$</p> <p>Use a number line so show equality of amount.</p>  <p>Bar Model – pupils need to understand multiplication facts to complete this model. E.g. $52 \div 4$</p> 	<p>Recall and use division facts from multiplication tables. E.g. $12 \div 6$</p>  <p>$12 \div 6 = 2$ $2 \times 6 = 12$</p>  <p>Bar Model – pupils need to understand multiplication facts to complete this model. E.g. $44 \div 4$</p> <p>44</p> 	<p>Short division E.g. $856 \div 4$</p>  <p>Bar Model – pupils need to understand multiplication facts to complete this model. E.g. $844 \div 4$</p>  <p>844</p>	
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Abstract		<p>E.g. $6 \div 2 = 3$</p> <table><tr><td>3</td><td>3</td></tr></table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3	<p>Divide two digits by one-digit numbers into groups. How many are in each group?</p> <p>E.g. $28 \div 7 = 4$</p> <p>Use a number line to divide using repeated subtraction.</p> <p>E.g. $10 \div 3$</p> <table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr></table> <p>Number Line Method</p>	1	2	3	4	5	0	1	2	3	4	5	6	7	8	9	10	<p>Pupils to be able to make sense of the place value counters and write calculations to show the process.</p> <p>$42 \div 3$</p> <p>$42 = 30 + 12$</p> <p>$30 \div 3 = 10$</p> <p>$12 \div 3 = 4$</p> <p>$10 + 4 = 14$</p> <p>E.g. $48 \div 4 =$</p> <table><tr><td>12</td></tr><tr><td>4 $\overline{)48}$</td></tr></table>	12	4 $\overline{)48}$	<p>Children to the calculation using the short division scaffold. When pupils begin to use short and long division, they should be confident with the abstract model.</p> <table><tr><td>123</td></tr><tr><td>5 $\overline{)615}$</td></tr></table> <table><tr><td></td><td>2</td><td>1</td><td>4</td></tr><tr><td>4</td><td>8</td><td>5</td><td>16</td></tr></table>	123	5 $\overline{)615}$		2	1	4	4	8	5	16	<p>Short division</p> <p>E.g. $856 \div 4$</p> <table><tr><td></td><td>2</td><td>1</td><td>4</td></tr><tr><td>4</td><td>8</td><td>5</td><td>16</td></tr></table> <p>Divide numbers by drawing on number facts and relationships.</p> <p>E.g. $436 \div 4$</p> <table><tr><td colspan="2">100 $\times$ 4</td><td>9 $\times$ 4</td></tr><tr><td>0</td><td>400</td><td>436</td></tr></table>		2	1	4	4	8	5	16	100 $\times$ 4		9 $\times$ 4	0	400	436	<p>Long division.</p> <p>E.g. $432 \div 12$</p> <p>E.g. $372 \div 15$</p> <table><tr><td></td><td></td><td>0</td><td>3</td><td>6</td></tr><tr><td>1</td><td>2</td><td>4</td><td>3</td><td>2</td></tr><tr><td></td><td>-</td><td>3</td><td>6</td><td>0</td></tr><tr><td></td><td></td><td></td><td>7</td><td>2</td></tr><tr><td></td><td>-</td><td></td><td>7</td><td>2</td></tr><tr><td></td><td></td><td></td><td></td><td>0</td></tr></table> <table><tr><td></td><td></td><td>2</td><td>4</td><td>r</td><td>1</td><td>2</td></tr><tr><td>1</td><td>5</td><td>3</td><td>7</td><td>2</td><td></td><td></td></tr><tr><td></td><td>-</td><td>3</td><td>0</td><td>0</td><td></td><td></td></tr><tr><td></td><td></td><td></td><td>7</td><td>2</td><td></td><td></td></tr><tr><td></td><td>-</td><td></td><td>6</td><td>0</td><td></td><td></td></tr><tr><td></td><td></td><td></td><td>1</td><td>2</td><td></td><td></td></tr></table>			0	3	6	1	2	4	3	2		-	3	6	0				7	2		-		7	2					0			2	4	r	1	2	1	5	3	7	2				-	3	0	0						7	2				-		6	0						1	2		
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